Overview of Robust Parameter Design

Consider the manufacture of a ball point pen.

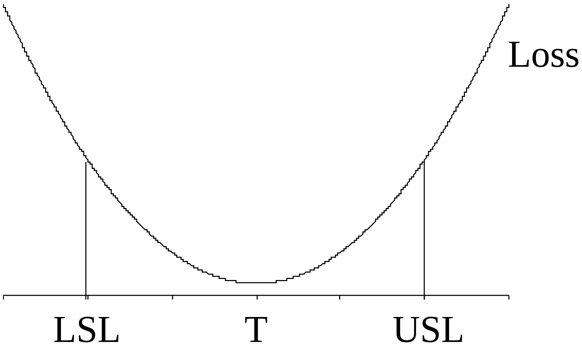
- **important characteristic is the fit between the barrel and the cap.**
- **barrel and the cap are produced by separate injection molding processes.**
- **How can we produce these barrels and caps such that the fit is "optimal"?**

What are the real issues in this problem?

Overview of Robust Parameter Design

The Japanese would view any part which does not achieve the target value as having some tangible loss of value.

$$\textbf{Loss} = k\,(y - T)^2$$



Overview of Robust Parameter Design

Impacts of such a philosophy

1. **should seek conditions which minimize the expected "loss"**
2. **must consider both the mean and the variance**

Overview of Robust Parameter Design

What is "high quality" performance?

- **Meets performance expectations**
  - **on the average, on target**
  - **with minimum variability**
- **Over the full range of process/use conditions**

Key idea: Robustness to conditions beyond the direct control of the manufacturer.

Overview of Robust Parameter Design

Fundamental to this approach are the concepts of

1. **control factors**

   factors which the experimenter can readily control.

2. **noise factors.**
   - **factors which the experimenter either cannot or will not directly control in the process**
   - **factors "move" randomly in actual process although they can be fixed for the experiment.**

Simple Example of Control and Noise Factors

Suppose we wish to develop a cake mix "robust" to customer use.

Possible control factors include

- **Number of eggs**
- **Amount of oil**
- **Amount of water**

Possible noise factors include

- **Baking time**
- **Baking temperature**
- **"Chef" is a divorced father**

Goal of parameter design: find the settings for the control factors which are most "robust" to the noise factors.

“Taguchi” Designs

Taguchi proposes “crossing”:

1. **a design for the control factors (inner or control array)**
2. **a design for the noise factors (outer or noise array)**

Each point of the inner array is replicated according to a design in the noise factors called the outer array.

Typically, these designs are “saturated” or “near-saturated”.

Saturated means the minimum number of unique combinations of the factors to estimate the proposed model.

"Taguchi" Designs

Consider our example.

The control factors are

- x_1: **number of eggs (-1 is 2; 1 is 3)**
- x_2: **amount of oil (-1 is 100 ml; 1 is 125 ml)**
- x_3: **amount of water (-1 is 400 ml; 1 is 500 ml).**

The noise factors are

- z_1: **baking time (-1 is 15 minutes; 1 is 21 minutes)**
  Recommended: 18
- z_2: **baking temperature (-1 is** $165 \deg C$**; 1 is** $185 \deg C$**)**
  Recommended: 175
- z_3: **divorced father (-1 is no; 1 is yes)**

An appropriate control array is a 2^{3-1} fraction or

x_1	x_2	x_3
-1	-1	-1
-1	1	1
1	-1	1
1	1	-1

Each of these settings is replicated by the outer array.

z_1	z_2	z_3
-1	-1	-1
-1	1	1
1	-1	1
1	1	-1

The resulting design consists of 4 $\times$ 4 or 16 runs and follows.

x_1	x_2	x_3	z_1	z_2	z_3
-1	-1	-1	-1	-1	-1
			-1	1	1
			1	-1	1
			1	1	-1
-1	1	1	-1	-1	-1
			-1	1	1
			1	-1	1
			1	1	-1
1	-1	1	-1	-1	-1
			-1	1	1
			1	-1	1
			1	1	-1
1	1	-1	-1	-1	-1
			-1	1	1
			1	-1	1
			1	1	-1

Why run the experiment in the noise factors?

Both the control and noise arrays are completely saturated

- **Cannot estimate control-by-control or noise-by-noise interactions.**
- **Can estimate all of the interactions between the control and noise factors!**
- **For Taguchi, the control-by-noise interactions are key to robustness.**

Why run the experiment in the noise factors?

- **We seek to find the settings in the control factors which are most "robust" to the noise factors.**
- **The levels of the noise factors reflect the expected range in values in the process/use.**
- **The natural consequence: Find conditions**
  - **that minimize variability**
  - **in presence of maxium noise.**

The “Combined Array” Method

The basic ideas underlying the “combined array” are

- **propose a single model in both the control and noise factors**
- **run a design specifically for the model proposed.**

In the process,

- **we can estimate some of the control by control interactions**
- **we can allocate our experimental resources more efficiently,**
- **in some cases, the resulting designs are significantly smaller.**
  (Usually, if we use fractional factorials for the crossed array, the combined array is about the same size.)

A "Combined" Array (2^{6-2} fractional factorial experiment)

x_1	x_2	x_3	z_1	z_2	z_3
-1	-1	-1	-1	-1	-1
1	-1	-1	-1	1	-1
-1	1	-1	-1	1	1
1	1	-1	-1	-1	1
-1	-1	1	-1	1	1
1	-1	1	-1	-1	1
-1	1	1	-1	-1	-1
1	1	1	-1	1	-1
-1	-1	-1	1	-1	1
1	-1	-1	1	1	1
-1	1	-1	1	1	-1
1	1	-1	1	-1	-1
-1	-1	1	1	1	-1
1	-1	1	1	-1	-1
-1	1	1	1	-1	1
1	1	1	1	1	1

Building on the Combined Array

The propagation of error formula due to the noise variables provides important insights:

$$y_i(\boldsymbol{x}_i, \boldsymbol{z}_i) = \beta_0 + \boldsymbol{f}(\boldsymbol{x}_i)'\boldsymbol{\beta} + \boldsymbol{z}_i'\gamma + \boldsymbol{f}(\boldsymbol{x}_i)'\boldsymbol{\Lambda}\boldsymbol{z}_i + \epsilon_i$$

- β_0 **is the** y**-intercept**
- $\boldsymbol{f}(\boldsymbol{x}_i)$ **is the vector of the control variables**
- $\boldsymbol{\beta}$ **is the vector of regression coefficients for the control effects**
- $\boldsymbol{z}$ **is the vector of noise factors,**
- γ **is the vector of regression coefficients for the noise factors**
- $\boldsymbol{\Lambda}$ **specifies the interactions**
- **the** ϵ**'s are random errors.**

Building on the Combined Array

Assume

- **The noise factors have been centered such that** $E(\boldsymbol{z}) = 0$
- **The variance-covariance structure of the noise variables is** $\boldsymbol{V}$ **where** $\boldsymbol{V}$ **is known.**

Then the process mean and process variance are

$$E\left[y_i(\boldsymbol{x}_i, \boldsymbol{z}_i)\right] = \beta_0 + \boldsymbol{f}(\boldsymbol{x}_i)'\boldsymbol{\beta}.$$

$$\textbf{var}\left[y_i(\boldsymbol{x}_i, \boldsymbol{z}_i)\right] = \left[\gamma + \boldsymbol{\Lambda}'\boldsymbol{f}(\boldsymbol{x}_i)\right]' \boldsymbol{V} \left[\gamma + \boldsymbol{\Lambda}'\boldsymbol{f}(\boldsymbol{x}_i)\right] + \sigma^2.$$

Building on the Combined Array

Note:

1. **If there are no interactions between the control and the noise factors, then the process variance is uniform over the region of interest.**
2. $$\frac{\partial y_i(\boldsymbol{x}_i, \boldsymbol{z}_i)}{\partial \boldsymbol{z}_i} = \boldsymbol{\gamma} + \boldsymbol{\Lambda}' \boldsymbol{f}(\boldsymbol{x}_i).$$

   Thus,

   $$\textbf{var}\left[y_i(\boldsymbol{x}_i, \boldsymbol{z}_i)\right] = \left(\frac{\partial y_i(\boldsymbol{x}_i, \boldsymbol{z}_i)}{\partial \boldsymbol{z}_i}\right)' \boldsymbol{V} \left(\frac{\partial y_i(\boldsymbol{x}_i, \boldsymbol{z}_i)}{\partial \boldsymbol{z}_i}\right) + \sigma^2.$$

   The process variance depends upon the slope of the response relative to the noise variables.

   This point helps to explain the importance of looking at interaction plots.

Adapting Robust Parameter Design

Complex systems, especially weapon systems, are the result of a large amount of experimentation.

Think sequential experimentation on steroids!

- **Components**
- **Sub-systems**
- **Assemblies of sub-systems**
- **Complete system**

Adapting Robust Parameter Design

Types of experiments conducted:

- **First principles computer**
- **Simulation**
- **Physical**

Experimental purposes/goals:

- **Exploration**
- **System characterization and performance prediction**
- **Meet contract specifications**
- **Optimize performance**
- **Confirm perfromance**

Computer Based Experimentation: First Principles

- **Very important in the initial experiments**
- **Based almost solely on underlying physics.**
- **Models are deterministic (do not reflect variability)**
- **General analysis approach:**
  - **Non-parametric smoothing models (kriging, Gaussian stochastic processes)**
  - **Hyper-Latin Square designs (uniform dispersion of the design runs)**
- **Primary Goal: Useful prediction model over entire region of possible operability.**
- **Utility depends on the model's fidelity.**
- **Models generally cannot provide causal inferences!**

Computer Based Experimentation: Simulation

- **Can be either deterministic or stochastic.**
- **Introduction of variability:**
  - **noise on the simulation outputs**
  - **noise on the simulation inputs (transmission of error)**
  - **combination of both**
- **Analysis approaches:**
  - **non-parametric smoothing (assumes a large region of interest)**
  - **low-order Taylor series (polynomial) models (assumes a focused region of interest)**
- **Polynomial models:**
  - **provide basis for causal inference**
  - **allow estimation of interactions**

Physical Experimentation

- **Always stochastic (assumes background noise)**
- **Almost always assumes localized approximations**
- **Physical experimentation provides best causal inferences!**
- **Interactions often are very important**
- **Eventually, all systems require some physical experimentation to evaluate and to confirm performance.**

System Performance Evaluation and Confirmation

- **Final stages of a long experimental process!**
- **Have developed a good understanding of factors that control system behavior.**
- **Primary goal: System effectiveness under actual use conditions.**
- **Remember Dr. Gilmore's report to the US Congress**
- **For weapon systems: The war-fighter should not have to experiment on the battlefield!**
- **Best approach is physical experimentation to the extent possible.**
- **High fidelity simulation experimentation can be effective but is open to criticism.**

Robust Parameter Design and System Performance Evaluation and Confirmation

- **Control factors: whatever controls the system behavior.**
- **Noise factors: factors outside the physical system that may impact the system in use**
  - **Environmental: weather, terrain.**
  - **Personnel as they use the system: training or lack thereof, "creative" use in the field**
  - **Weapons: adversary tactics**
- **Important differences:**
  - **Most noise factors in robust parameter design are continuous: time, temperature, humidity**
  - **For many complex systems, the noise factors are categorical with many levels.**

Issues with categorical noise factors

- **Classical Taguchi robust parameter design assumes either two or three levels for all factors, especially the noise.**
- **Classic Taguchi designs are two-level and three-level Plackett-Burman designs.**
- **Number of terms in model determines design size.**
- **Number of interactions increase with number of levels**
  - **control factors (often continuous with two or three levels)**
  - **system noise effects (often categorical with many levels)**
  - **Let a be the number of levels for a single control factor**
  - **Let b be the number of levels for a single noise factor.**
  - **Their total number of interaction terms is $(a-1) \times (b-1)$.**

Issues with categorical noise factors

Consider an air-to-surface missile system:

- **Control or system factor: range (continuous)**
- **Noise or environmental factor: weather (discrete)**

Levels for each factor:

- **Range (x_1): Three - 10, 15, 20 miles**
- **Weather (z_1): Seven - sunny and clear, partially cloudy, cloudy, completely overcast, light rain, heavy rain, thunderstorm**
- **Total number of interaction terms: $(3-1) \times (7-1) = 14$.**
  - **Six terms involve x_1 with the environmental levels**
  - **Six terms involve x_1^2 with the environmental levels.**

Algorithmic designs

The cost of a single test run often requires an experiment with as few runs as possible.

The classical experimental design approach is a full factorial experiment.

For the one control and one noise factor experiment, the total number of runs is $a \times b$.

The absolute minimum number of test runs for an experiment is the total number of terms to be estimated.

Typically, to reduce the experimental cost, the analysts a priori determine that some interactions are "not interesting."

Algorithmic designs

The situation gets much worse as we add more control and noise factors.

Let ℓ_j be the number of proposed levels for the j^{th} factor (both control and noise).

The classic factorial experiment is every unique combination of these levels.

Suppose we have a total of k factors.

The resulting total number of runs is $\ell_1 \times \ell_2 \times \ldots \times \ell_k = \prod_{j=1}^{k} \ell_j$.

This design can estimate all possible interactions, many of which are not interesting.

Algorithmic designs

Critical question: How do we select a "good" subset from all of these possible combinations?

Popular answer: Algorithmic (optimal) designs.

- **The analyst picks a statistically based criterion to select the design runs.**
  - **"D-optimality" - minimizes the determinant of the information matrix**
  - **"I-optimality" - minimizes the average prediction variance**
- **Determine the total number of runs desired for the experiment**
- **Specify the "model" - think which factors and which interactions to estimate**
- **Use computer code based on a good algorithm to find the "optimal" design in terms of the criterion.**

Algorithmic designs

- **Traditional focus of algorithmic design:**
  - **When choosing the model, main effects more important than interactions**
  - **Equal importance to all terms in the model, even those already well-understood.**
- **Critical to Evaluation and Confirmation: Identifying the critical interactions!**
  - **Identify problems in system use (both operational and late stage developmental testing)**
  - **Learn to mitigate these problems (sooner the better!)**
  - **For these tests, the critical interactions are more important than the main effects!**

Algorithmic designs

Major issues:

- **Must tailor the evaluation - confirmation design to accommodate the critical interactions.**
- **Need to modify the most common computer code for selecting the design. (research issue)**
- **Must plan experiment to quantify (ideally minimize) uncertainty in performance given budget constraints.**
- **Final evaluation must express the statistical uncertainty of success (even if not particularly good)!**

Take-Aways

- **The ultimate goals for the testing of complex systems:**
  - **meet performance expectations over a wide variety of operating conditions**
  - **identify problematic operating conditions**
  - **learn proper mitigation strategies.**
- **Must adapt standard design and analysis of experiments methods.**
- **Robust parameter design provides an attractive approach.**
- **Does need some adaptation.**