Reliability Fundamentals and Analysis Lessons Learned

Maj Brian Stone
Mr. Dan Telford
HQ AFOTEC
Release Date: 12 Mar 18
Overview

• Reliability Fundamentals
  − Definitions
  − Graphs of a failure process
  − Censored Vs. uncensored data

• Lessons learned from reliability analysis
  − Case Study #1: F-22 infant mortality
  − Case Study #2: F-22 weapon system reliability
  − Case Study #3: RQ-1 Predator failure analysis
Reliability Fundamentals
Definitions

• Statistical methods are used to determine part reliability using part failure data

• **Failure** - The inability of a component or system to perform its intended function for a specified time under specified environment conditions

• **Reliability** - The probability that a component or system will perform its required function satisfactorily for a specified period of time when used under stated operating conditions

• **Maintainability** - The probability that a failed component or system will be restored or repaired to a specified condition within a period of time when maintenance is performed in accordance with pre-specified procedures

• **Availability** - The probability that a component or system is performing its required function at a given point in time when used under stated operating conditions
A Word About “Time”

• Unit of measurement for “time”
  − Hours, minutes, seconds
  − Cycles
  − Miles
  − Rounds
  − Etc.

• Whatever “life units” cause the system to age
  − Thermal cycles
  − Take offs and landings
  − Flight time
A failure process, represented by a random variable $T$ (time to failure) may be uniquely characterized by any of the following four functions:

- Cumulative Distribution Function: $F(t)$
- Probability Density Function: $f(t)$
- Reliability Function: $R(t)$
- Hazard Function: $h(t)$
• Consider the following data for failures by unit time
• The ratio of the number of failures between time \([t-1, t)\) and the total number of failures can be considered the probability of failing between time \([t-1, t)\)
• The graph represents the probability distribution function (pdf) of failure times
Cumulative Distribution Function

- At each time step we can calculate the cumulative failures just before time $t$
- The ratio of the number of the cumulative failures before time $t$ and the total number of failures at each time step is the Cumulative Distribution Function (CDF), aka $F(t)$
- The graph represents the Cumulative Distribution Function (CDF) of failure times
The proportion of systems that haven’t failed by each time step can be calculated using $F(t)$

- This is called the reliability function $R(t)$ or the survivor function
- We compute this value by subtracting the number of failures before time $t$ from the total number of failures
- In general this is computed as $1 - F(t)$
Hazard Function

• Recall that \( f(t) \) is the *unconditional* probability that a unit will fail in the interval \((t-1, t)\)

• The hazard function \( h(t) \) is the *conditional* probability that the unit will fail in the interval \((t-1, t)\), given that it has survived until time \( t-1 \)

• The hazard function is computed as \( \frac{f(t)}{R(t)} \)

| \( h(t) \) | \( P(t-1 \leq T < t | T \geq t-1) \) |
|---|---|
| \( t \) (Time) | Num | Denom | Ratio |
| 1 | 0 | 100 | 0.00 |
| 2 | 0 | 100 | 0.00 |
| 3 | 2 | 100 | 0.02 |
| 4 | 13 | 98 | 0.13 |
| 5 | 20 | 85 | 0.24 |
| 6 | 13 | 65 | 0.20 |
| 7 | 14 | 52 | 0.27 |
| 8 | 13 | 38 | 0.34 |
| 9 | 13 | 25 | 0.52 |
| 10 | 6 | 12 | 0.50 |
| 11 | 2 | 6 | 0.33 |
| 12 | 2 | 4 | 0.50 |
| 13 | 2 | 2 | 1.00 |
Exponential Distribution

• The exponential distribution is often used as a probability distribution function $f(t)$

$$f(t) = \lambda e^{-\lambda t}$$

• The CDF $F(t)$ is the integral of $f(t)$

$$F(t) = \int_0^t f(t) = \int_0^t \lambda e^{-\lambda t} = 1 - e^{-\lambda t}$$

• The Reliability Function $R(t)$ is $1 - F(t)$

$$R(t) = 1 - F(t) = 1 - (1 - e^{-\lambda t})$$

$$R(t) = e^{-\lambda t}$$

• The Hazard Function $h(t)$ is

$$h(t) = \frac{f(t)}{R(t)}$$

$$\frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

• For the exponential pdf, the Hazard Function is constant!
Exponential

**Exponential pdf**

**Exponential cdf**

**Exponential Survival (Reliability)**

**Exponential Hazard Function**
Censored Data

• Reliability data is often censored
• Different types of censoring:
  – Right-censored data – failure times exceed the termination of the test
  – Left-censored data – failure times occur before the first inspection
  – Interval censored data – failure times occur between inspection intervals
  – Random censoring – a component may be damaged during a test
Uncensored Vs. Censored Data

Uncensored Data
Test to failure (data is not truncated)
Uncensored Vs. Censored Data

Censored Data
Test to cut-off time
(data is truncated)
Case Study #1

F-22: Infant Mortality
Case Study #1

• The system under test was the F-22 Raptor (3 aircraft) from the factory

• Wing utilized aircraft with a constant flying schedule
  – 2-3 sorties/weekday across 3 aircraft
  – Average Sortie Duration = 1.5 hours

• The Initial Operational Test and Evaluation (IOT&E) report provided the following reliability measure:
  \[
  \frac{Total \ MX \ Actions}{Total \ Flying \ Hours} = \frac{2073}{573} = 3.62 \ MX \ Actions \ Per \ Flying \ Hour
  \]

• The measure threshold was 3.5 maintenance (MX) actions per flying hour

• There was anecdotal evidence of initially frequent MX repairs that seemed to diminish over time (6 months)
Case Study #1

- Post-report analysis revealed additional insights
- The high MX actions per flying hour was likely due to infant mortality
Case Study #1

• Typical “bathtub” Rate of Occurrence of Failures (ROCOF) curve for repairable systems

Failures Caused by:
Manufacturing defects
Flaws
Defective parts

Failures Caused by:
Fatigue
Corrosion
Aging

Failures Caused by:
“Acts of God”
Human error
Chance events
Case Study #1

• Summary
  - Initial (1-2 months) repair rate of 5-6 MX actions per flying hour
  - At end of test (6 months) repair rate was 2-3 MX actions per flying hour

• Lessons Learned
  - Don’t assume the system’s ROCOF curve is in a steady state
  - Look at your data
  - Understand the context of your data
Case Study #2

F-22: Weapon System Reliability
(Tell the Whole Story)
Case Study #2

• **Weapon System Reliability (WSR)**
  - Measures the probability that a system will perform satisfactorily for a given mission time when used under specified conditions

• The F-22 report provided the standard AFPAM 63-128 WSR metric:
  - Successful Missions / Total Missions
  - WSR was reported as 250/300 = 83.3%

• Anecdotal stories
  - If the aircraft had no failures within the first 15 minutes of flight then a successful sortie was very likely
  - The implication was reliability changed over the course of a sortie
  - Indicative of a non-exponential reliability function (non-constant hazard rate)
  - Average sortie duration (ASD) was 1.5 hours
Case Study #2

- Some JMP analysis corroborated the anecdotal evidence
- The majority of failures took place before 20 minutes of flight time
Case Study #2

• **Summary**
  - Standard reliability metric was reported (WSR)
  - Anecdotal evidence led to a more thorough analysis

• **Lessons Learned**
  - Visualize your data
  - Do not assume weapon system reliability is constant over the mission
Case Study #3

RQ-1 Predator: Problems with Assumption of Exponential Distribution
Case Study #3

• A failure density distribution that has a constant failure rate has an exponential reliability distribution

• Many systems exhibit constant failure rates, and the exponential reliability distribution is the simplest to analyze

• Suppose we tested 500 systems for 2000 hours each and observed 100 failures; first calculate the Mean Time Between Failures (MTBF)

\[
MTBF = \frac{Total\ Time}{\#\ Failures} = \frac{2000 \times 500}{100} = 10,000\ hours
\]

• Let the reliability at time \( t \) be

\[
R(t) = e^{-\lambda t}\ where\ \lambda = \frac{1}{MTBF}
\]

• The reliability at 10,000 hours:

\[
R(t) = e^{-\lambda t} = e^{-\frac{1}{10000} \times 10000} = 0.37
\]

Figure 3. The Exponential Reliability Function whose MTBF is 10,000 hours.
Predator Data

- Test focus: Comm Subsystem
  - Failure is “Lost Link”
  - Always a reboot
  - Reloaded all parameters
- Reliability assumed exponential
  \[ MTBF = \frac{\text{Total Time}}{\text{Total Failures}} \]
  \[ MTBF = \frac{29 \times 30}{53} = 16.42 \text{ hours} \]
- Suppose we are concerned with reliability at 4 hours:
  \[ R(t) = e^{-\lambda t} = e^{\frac{-4}{16.42}} = 0.78 \]
- Problems:
  - Did not treat data as censored
  - Assumed the exponential reliability function
Predator Data

• If we compute the times between failures and account for censoring, the data looks like this
• An “Event Plot” from JMP analysis
• Does it seem reasonable that the reliability at 4 hours is 78% (18/82 failures by 4 hours?)
• What other distributions can be used to model the reliability function?
The lognormal distribution is a common model for failure times.
The Weibull distribution can be used to model failure-time data with a decreasing or an increasing hazard function.
Case Study #3

- JMP lets us compare the fit of various reliability distributions to our failure time data.
- This is the fit for an exponential distribution with $\lambda = 16.42$. 
Case Study #3

- On the left is the fit for a lognormal distribution with $\mu = 2.446$ and $\sigma^2 = 0.773$.
• On the left is the fit for a Weibull distribution with $\alpha = 15.448$ and $\beta = 1.738$

Weibull Distribution

Exponential Distribution
Case Study #3

- The JMP “Model Comparisons” output shows that the Weibull has the best fit overall.
- However, which reliability function fits the data best between 0 to 10 hours?
- Using the lognormal reliability function, the reliability at 4 hours is 0.915.
- Recall we computed the reliability at 4 hours to be 0.78 using the exponential function.
Case Study #3

• Summary
  – The assumption that RQ-1 Predator failures had an Exponential reliability function was incorrect
  – The Lognormal distribution was a much better model for the reliability curve

• Lessons Learned
  – Visualize your data
  – Consider multiple models for reliability curves
Questions